Northern Border University Faculty of Engineering First Semester 1436/1437, Mid Term Exam		Subject: Operations Research IE311 Date: 16/1/1437, Time allowed: 1.5 Hrs Total Marks: 10 Points Instructors: Dr. **Mohamed Mostafa**

Solve the Following Problems

1) Solve the Following LPP by graphical method. **(2)**

Minimize: $Z = 4\,x_1 + 3\,x_2$

Subject to the constraints

$$2\,x_1 + x_2 \geq 10$$
$$-3\,x_1 + 2\,x_2 \leq 6$$
$$x_1 + x_2 \geq 6$$
And $x_1, x_2 \geq 0$

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2) A manufacturing company engaged in producing three types of products: A, B and C. The production department daily produces component sufficient to make 50 units of A, 25 units of B and 30 units of C. The management is confronted with problem of optimizing the daily production of products in assembly department where only 100 man-hours are available daily to assemble the products. The following additional information is available.

Type of product	Profit contribution per unit of product (SR)	Assembly time per product (hrs)
A	12	0.8
B	20	1.7
C	45	2.5

The company has a daily order commitment for 20 units of product A and total of 15 units of B and C products. **Formulates** this problem as an LP model so as to maximize the total profit. **(3)**

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3) Solve the following LPP by Simplex Method **(5)**

Maximize $Z = 2\,x_1 + x_2 + x_3$

Subject to the constraints

$$4\,x_1 + 6x_2 + 3x_3 \leq 8$$
$$3\,x_1 - 6x_2 - 4x_3 \leq 1$$
$$2\,x_1 + 3x_2 - 5x_3 \geq 4$$
$$\text{and } x_1, x_2, x_3 \geq 0$$

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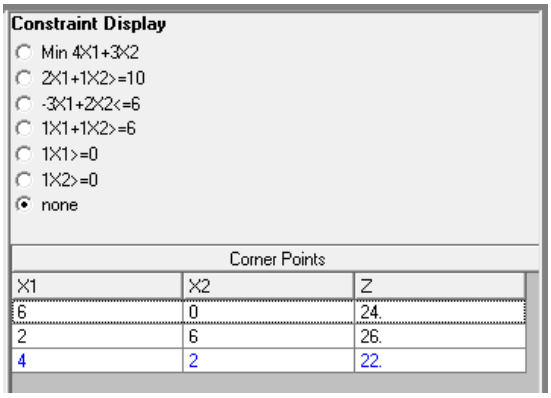
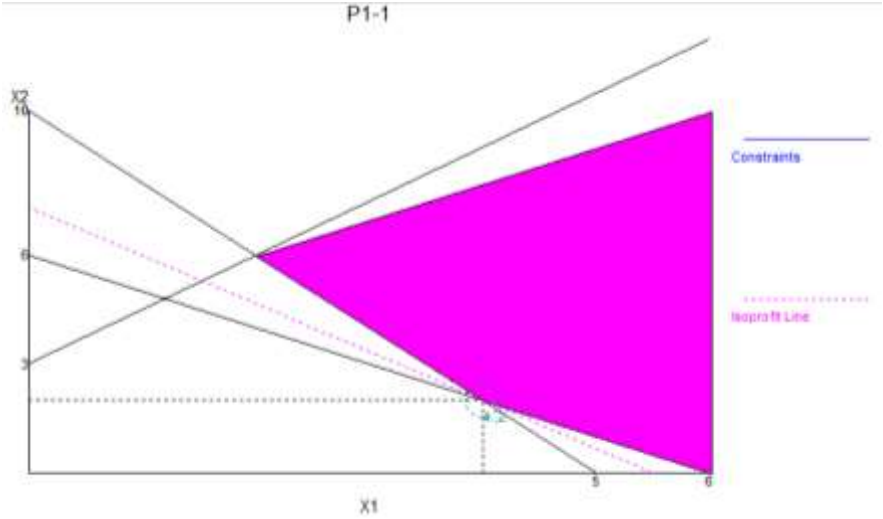
Model Answer of Mid Term Exam Group [1]

Solution of Problem #1





Solution of Problem #2

LP model formulation: the data of the problem is summarized as follows:

Resources / Constraints	product type A	B	C	total
Production capacity (units)	50	25	30	
Man hours per unit	0.8	1.7	2.5	100
Order commitment unit		20	15	
Profit contribution (Rs./unit)	12	20	45	

Decision variables: let x1, x2, x3=numbers of units of products A, B and C to be produced respectively

The LP model

Maximize (total profit) Z = 12x1 + 20x2 + 45x3

Subject to the constraints

(a) labor and material constraints

0.8x1+1.7x2+2.5x3 =<100

x1 =<50

x2 =<25

x3=<30

(b) order commitment constraints

x1 >=20

x2 +x3 >=15

x1,x2,x3>=0

Solution of Problem #3

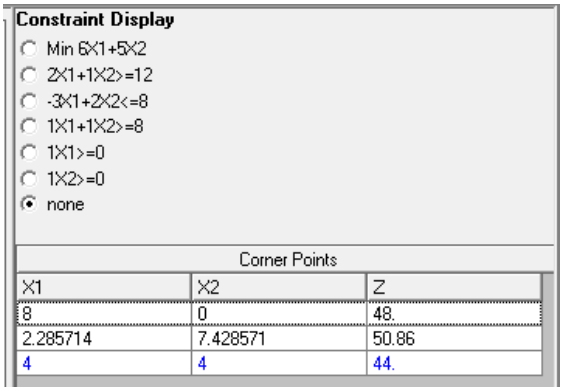
Variable	Status	Value
X1	Basic	1.2857
X2	Basic	.4762
X3	Basic	0
slack 1	NONBasic	0
slack 2	NONBasic	0
surplus 3	NONBasic	0
surplus 4	Basic	1.2857
surplus 5	Basic	.4762
Optimal Value (Z)		3.0476

Note
Multiple optimal solutions exist

Cj	Basic Variables	2 X1	1 X2	1 X3	0 slack 1	0 slack 2	0 artfcl 3	0 surplus 3	0 artfcl 4	0 surplus 4	0 artfcl 5	0 surplus 5	Quantity
0	slack 1	4	6	3	1	0	0	0	0	0	0	0	8
0	slack 2	3	-6	-4	0	1	0	0	0	0	0	0	1
0	artfcl 3	2	3	-5	0	0	1	-1	0	0	0	0	4
0	artfcl 4	1	0	0	0	0	0	0	1	-1	0	0	0
0	artfcl 5	0	1	0	0	0	0	0	0	0	1	-1	0
Iteration 2													
	cj-zj	0	4	-5	0	0	0	-1	-3	2	0	-1	
0	slack 1	0	6	3	1	0	0	0	-4	4	0	0	8
0	slack 2	0	-6	-4	0	1	0	0	-3	3	0	0	1
0	artfcl 3	0	3	-5	0	0	1	-1	-2	2	0	0	4
2	X1	1	0	0	0	0	0	0	1	-1	0	0	0
0	artfcl 5	0	1	0	0	0	0	0	0	0	1	-1	0
Iteration 3													
	cj-zj	0	0	-5	0	0	0	-1	-3	2	-4	3	
0	slack 1	0	0	3	1	0	0	0	-4	4	-6	6	8
0	slack 2	0	0	-4	0	1	0	0	-3	3	-6	6	1
0	artfcl 3	0	0	-5	0	0	1	-1	-2	2	-3	3	4
2	X1	1	0	0	0	0	0	0	1	-1	0	0	0
1	X2	0	1	0	0	0	0	0	0	0	1	-1	0
Iteration 4													
	cj-zj	0	0	-6.5	-0.5	0	0	-1	-1	0	-1	0	
0	surplus 5	0	0	0.5	0.1667	0	0	0	-0.6667	0.6667	-1	1	1.3333
0	slack 2	0	0	-1	1	1	0	0	-7	7	0	0	9
0	artfcl 3	0	0	-6.5	-0.5	0	1	-1	0	0	0	0	0
2	X1	1	0	0	0	0	0	0	1	-1	0	0	0
1	X2	0	1	0.5	0.1667	0	0	0	-0.6667	0.6667	0	0	1.3333
Iteration 5													
	cj-zj	0	0	0	0	0	-1	0	-1	0	-1	0	
0	surplus 5	0	0	0	0.1282	0	0.0769	-0.0769	-0.6667	0.6667	-1	1	1.3333
0	slack 2	0	0	0	1.0769	1	-0.1538	0.1538	-7	7	0	0	9
1	X3	0	0	1	0.0769	0	-0.1538	0.1538	0	0	0	0	0
2	X1	1	0	0	0	0	0	0	1	-1	0	0	0
1	X2	0	1	0	0.1282	0	0.0769	-0.0769	-0.6667	0.6667	0	0	1.3333
Iteration 6													
	cj-zj	0	0	0	-0.2051	0	0.0769	-0.0769	-1.3333	1.3333	0	0	
0	surplus 5	0	0	0	0.1282	0	0.0769	-0.0769	-0.6667	0.6667	-1	1	1.3333
0	slack 2	0	0	0	1.0769	1	-0.1538	0.1538	-7	7	0	0	9
1	X3	0	0	1	0.0769	0	-0.1538	0.1538	0	0	0	0	0
2	X1	1	0	0	0	0	0	0	1	-1	0	0	0
1	X2	0	1	0	0.1282	0	0.0769	-0.0769	-0.6667	0.6667	0	0	1.3333
Iteration 7													
	cj-zj	0	0	0	-0.4103	-0.1905	0.1962	-0.1962	0	0	0	0	
0	surplus 5	0	0	0	0.0256	-0.0952	0.0916	-0.0916	0	0	-1	1	0.4762
0	surplus 4	0	0	0	0.1538	0.1429	-0.022	0.022	-1	1	0	0	1.2857
1	X3	0	0	1	0.0769	0	-0.1538	0.1538	0	0	0	0	0
2	X1	1	0	0	0.1538	0.1429	-0.022	0.022	0	0	0	0	1.2857
1	X2	0	1	0	0.0256	-0.0952	0.0916	-0.0916	0	0	0	0	0.4762

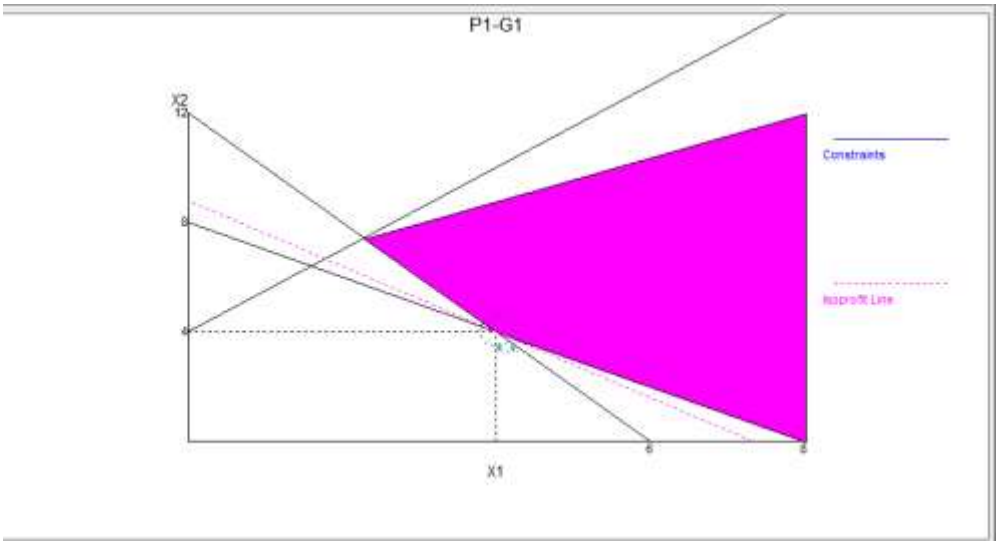
Model Answer of Mid Term Exam Group [2]

Solution of Problem #1



Corner Points

X1	X2	Z
8	0	48.
2.285714	7.428571	50.86
4	4	44.



Solution of Problem #2

Resources / Constraints	product type A	B	C	total
Production capacity (units)	50	25	30	
Man hours per unit	0.8	1.7	2.5	100
Order commitment unit		20	15	
Profit contribution (Rs./unit)	14	22	47	

Decision variables: let x1, x2, x3=numbers of units of products A, B and C to be produced respectively

The LP model

Maximize (total profit) Z = 14x1 + 22x2 + 47x3

Subject to the constraints

(c) labor and material constraints

0.8x1+1.7x2+2.5x3 =<100

x1 =<50

x2 =<25

x3=<30

(d) order commitment constraints

x1 >=20

x2 +x3 >=15

x1, x2, x3>=0

Solution of Problem #3

Equation form
Max 4X1 + 3X2 + 3X3
4X1 + 6X2 + 3X3 <= 10
3X1 --6X2 --4X3 <= 3
2X1 + 3X2 --5X3 >= 6
X1 >= 0
X2 >= 0

There is no feasible solution

P3-2 Solution

Cj	Basic Variables	4 X1	3 X2	3 X3	0 slack 1	0 slack 2	0 artfcl 3	0 surplus 3	0 artfcl 4	0 surplus 4	0 artfcl 5	0 surplus 5	Quantity
Iteration 1													
	cj-zj	3	4	-5	0	0	0	-1	0	-1	0	-1	
0	slack 1	4	6	3	1	0	0	0	0	0	0	0	10
0	slack 2	3	-6	-4	0	1	0	0	0	0	0	0	3
0	artfcl 3	2	3	-5	0	0	1	-1	0	0	0	0	6
0	artfcl 4	1	0	0	0	0	0	0	1	-1	0	0	0
0	artfcl 5	0	1	0	0	0	0	0	0	0	1	-1	0
Iteration 2													
	cj-zj	0	4	-5	0	0	0	-1	-3	2	0	-1	
0	slack 1	0	6	3	1	0	0	0	-4	4	0	0	10
0	slack 2	0	-6	-4	0	1	0	0	-3	3	0	0	3
0	artfcl 3	0	3	-5	0	0	1	-1	-2	2	0	0	6
4	X1	1	0	0	0	0	0	0	1	-1	0	0	0
0	artfcl 5	0	1	0	0	0	0	0	0	0	1	-1	0
Iteration 3													
	cj-zj	0	0	-5	0	0	0	-1	-3	2	-4	3	
0	slack 1	0	0	3	1	0	0	0	-4	4	-6	6	10
0	slack 2	0	0	-4	0	1	0	0	-3	3	6	-6	3
0	artfcl 3	0	0	-5	0	0	1	-1	-2	2	-3	3	6
4	X1	1	0	0	0	0	0	0	1	-1	0	0	0
3	X2	0	1	0	0	0	0	0	0	0	1	-1	0
Iteration 4													
	cj-zj	0	0	-6.5	-0.5	0	0	-1	-1	0	-1	0	
0	surplus 5	0	0	0.5	0.1667	0	0	0	-0.6667	0.6667	-1	1	1.6667
0	slack 2	0	0	-1	1	1	0	0	-7	7	0	0	13
0	artfcl 3	0	0	-6.5	-0.5	0	1	-1	6	0	0	0	1.8
4	X1	1	0	0	0	0	0	0	1	-1	0	0	0
3	X2	0	1	0.5	0.1667	0	0	0	-0.6667	0.6667	0	0	1.6667