Sheet 2-Chapter 2 PROBLEMS

An Introduction to Linear Programming

11. Solve the following linear program using the graphical solution procedure:

$$\begin{aligned} \text{Max} \quad & 5A + 5B \\ \text{s.t.} \quad & \\ & 1A \leq 100 \\ & 1B \leq 80 \\ & 2A + 4B \leq 400 \\ & A, B \geq 0 \end{aligned}$$

12. Consider the following linear programming problem:

$$\begin{aligned} \text{Max} \quad & 3A + 3B \\ \text{s.t.} \quad & \\ & 2A + 4B \leq 12 \\ & 6A + 4B \leq 24 \\ & A, B \geq 0 \end{aligned}$$

a. Find the optimal solution using the graphical solution procedure.
b. If the objective function is changed to 2A + 6B, what will the optimal solution be?
c. How many extreme points are there? What are the values of A and B at each extreme point?

13. Consider the following linear program:

$$\begin{aligned} \text{Max} \quad & 1A + 2B \\ \text{s.t.} \quad & \\ & 1A \leq 5 \\ & 1B \leq 4 \\ & 2A + 2B = 12 \\ & A, B \geq 0 \end{aligned}$$

a. Show the feasible region.
b. What are the extreme points of the feasible region?
c. Find the optimal solution using the graphical procedure.

18. For the linear program

$$\begin{aligned} \text{Max} \quad & 4A + 1B \\ \text{s.t.} \quad & \\ & 10A + 2B \leq 30 \\ & 3A + 2B \leq 12 \\ & 2A + 2B \leq 10 \\ & A, B \geq 0 \end{aligned}$$

- **a.** Write this problem in standard form.
- **b.** Solve the problem using the graphical solution procedure.
- **c.** What are the values of the three slack variables at the optimal solution?

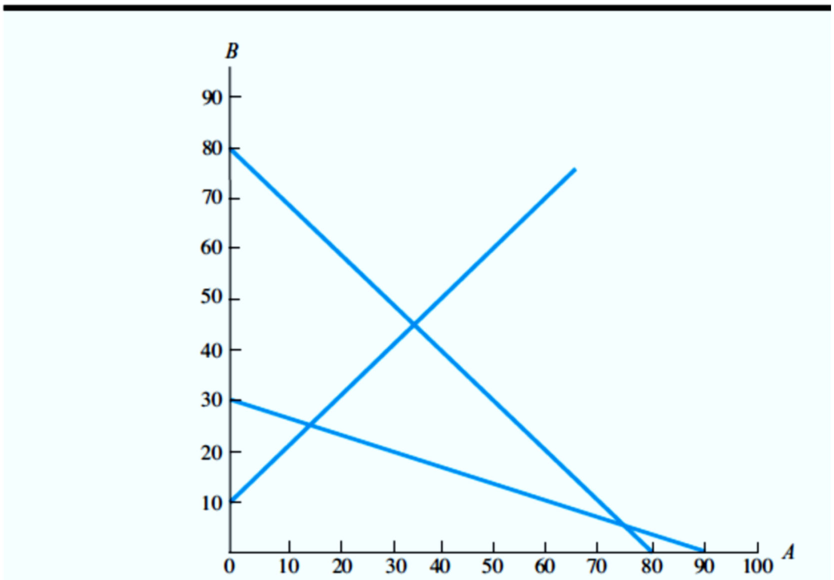
21. Consider the following linear program:

$$\begin{aligned} \text{Max} \quad & 2A + 3B \\ \text{s.t.} \quad & \\ & 5A + 5B \leq 400 \quad \text{Constraint 1} \\ & -1A + 1B \leq 10 \quad \text{Constraint 2} \\ & 1A + 3B \geq 90 \quad \text{Constraint 3} \\ & A, B \geq 0 \end{aligned}$$

Figure 2.22 shows a graph of the constraint lines.

- **a.** Place a number (1, 2, or 3) next to each constraint line to identify which constraint it represents.
- **b.** Shade in the feasible region on the graph.
- **c.** Identify the optimal extreme point. What is the optimal solution?
- **d.** Which constraints are binding? Explain.
- **e.** How much slack or surplus is associated with the nonbinding constraint?

FIGURE 2.22 GRAPH OF THE CONSTRAINT LINES FOR EXERCISE 21



22. Reiser Sports Products wants to determine the number of All-Pro (A) and College (C) footballs to produce in order to maximize profit over the next four-week planning horizon. Constraints affecting the production quantities are the production capacities in three departments: cutting and dyeing; sewing; and inspection and packaging. For the four-week planning period, 340 hours of cutting and dyeing time, 420 hours of sewing time, and 200 hours of inspection and packaging time are available. All-Pro footballs provide a profit of \$5 per unit and College footballs provide a profit of \$4 per unit. The linear programming model with production times expressed in minutes is as follows:

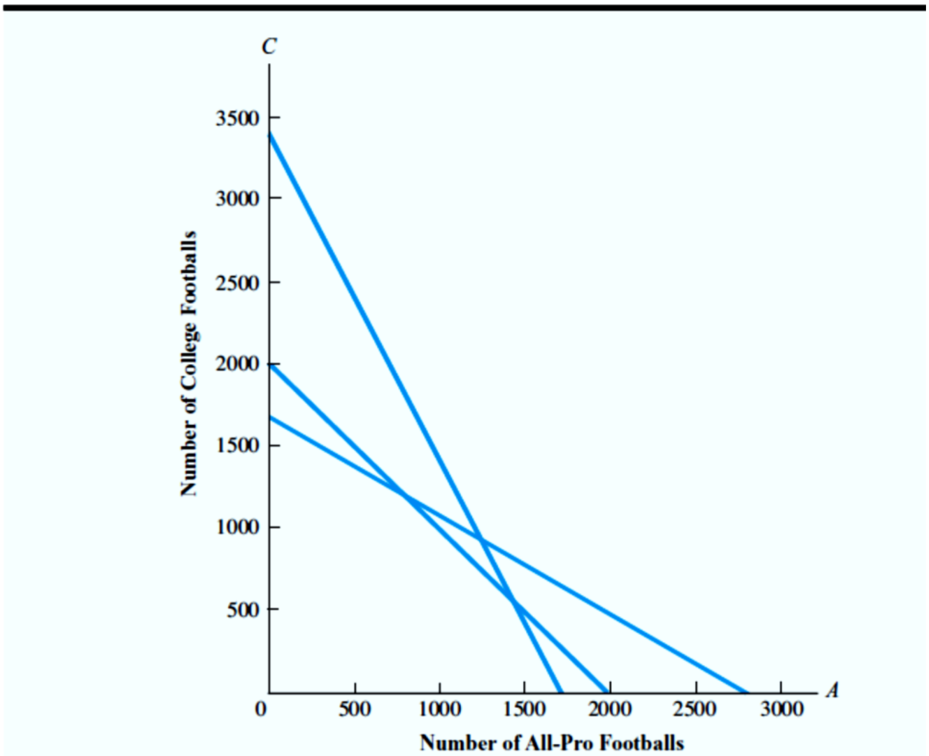
$$
\begin{array}{lrcll}
\text{Max} & 5A + 4C & & & \\
\text{s.t.} & & & & \\
 & 12A + 6C & \leq & 20{,}400 & \text{Cutting and dyeing} \\
 & 9A + 15C & \leq & 25{,}200 & \text{Sewing} \\
 & 6A + 6C & \leq & 12{,}000 & \text{Inspection and packaging} \\
 & A, C \geq 0 & & &
\end{array}
$$

A portion of the graphical solution to the Reiser problem is shown in Figure 2.23.

a. Shade the feasible region for this problem.

b. Determine the coordinates of each extreme point and the corresponding profit. Which extreme point generates the highest profit?

FIGURE 2.23 PORTION OF THE GRAPHICAL SOLUTION FOR EXERCISE 22



- **c.** Draw the profit line corresponding to a profit of $4000. Move the profit line as far from the origin as you can in order to determine which extreme point will provide the optimal solution. Compare your answer with the approach you used in part (b).
- **d.** Which constraints are binding? Explain.
- **e.** Suppose that the values of the objective function coefficients are $4 for each All-Pro model produced and $5 for each College model. Use the graphical solution procedure to determine the new optimal solution and the corresponding value of profit.

23. Embassy Motorcycles (EM) manufacturers two lightweight motorcycles designed for easy handling and safety. The EZ-Rider model has a new engine and a low profile that make it easy to balance. The Lady-Sport model is slightly larger, uses a more traditional engine, and is specifically designed to appeal to women riders. Embassy produces the engines for both models at its Des Moines, Iowa, plant. Each EZ-Rider engine requires 6 hours of manufacturing time and each Lady-Sport engine requires 3 hours of manufacturing time. The Des Moines plant has 2100 hours of engine manufacturing time available for the next production period. Embassy's motorcycle frame supplier can supply as many EZ-Rider frames as needed. However, the Lady-Sport frame is more complex and the supplier can only provide up to 280 Lady-Sport frames for the next production period.

Final assembly and testing requires 2 hours for each EZ-Rider model and 2.5 hours for each Lady-Sport model. A maximum of 1000 hours of assembly and testing time are available for the next production period. The company's accounting department projects a profit contribution of $2400 for each EZ-Rider produced and $1800 for each Lady-Sport produced.

- **a.** Formulate a linear programming model that can be used to determine the number of units of each model that should be produced in order to maximize the total contribution to profit.
- **b.** Solve the problem graphically. What is the optimal solution?
- **c.** Which constraints are binding?

24. Kelson Sporting Equipment, Inc., makes two different types of baseball gloves: a regular model and a catcher's model. The firm has 900 hours of production time available in its cutting and sewing department, 300 hours available in its finishing department, and 100 hours available in its packaging and shipping department. The production time requirements and the profit contribution per glove are given in the following table:

Model	Cutting and Sewing	Finishing	Packaging and Shipping	Profit/Glove
	Production Time (hours)			
Regular model	1	½	⅛	$5
Catcher's model	³⁄₂	⅓	¼	$8

Assuming that the company is interested in maximizing the total profit contribution, answer the following:

a. What is the linear programming model for this problem?

b. Find the optimal solution using the graphical solution procedure. How many gloves of each model should Kelson manufacture?

c. What is the total profit contribution Kelson can earn with the given production quantities?

d. How many hours of production time will be scheduled in each department?

e. What is the slack time in each department?

31. Consider the following linear program:

$$\begin{aligned} \text{Min} \quad & 3A + 4B \\ \text{s.t.} \quad & \\ & 1A + 3B \geq 6 \\ & 1A + 1B \geq 4 \\ & A, B \geq 0 \end{aligned}$$

Identify the feasible region and find the optimal solution using the graphical solution procedure. What is the value of the objective function?

34. Consider the following linear program:

$$\begin{aligned} \text{Min} \quad & 2A + 2B \\ \text{s.t.} \quad & \\ & 1A + 3B \leq 12 \\ & 3A + 1B \geq 13 \\ & 1A - 1B = 3 \\ & A, B \geq 0 \end{aligned}$$

a. Show the feasible region.

b. What are the extreme points of the feasible region?

c. Find the optimal solution using the graphical solution procedure.