Sheet 3-Chapter 3 PROBLEMS

Linear Programming: Sensitivity Analysis and Interpretation of Solution

1. Consider the following linear program:

$$\begin{aligned} \text{Max} \quad & 3A + 2B \\ \text{s.t.} \quad & \\ & 1A + 1B \le 10 \\ & 3A + 1B \le 24 \\ & 1A + 2B \le 16 \\ & A, B \ge 0 \end{aligned}$$

a. Use the graphical solution procedure to find the optimal solution.

b. Assume that the objective function coefficient for *A* changes from 3 to 5. Does the optimal solution change? Use the graphical solution procedure to find the new optimal solution.

c. Assume that the objective function coefficient for *A* remains 3, but the objective function coefficient for *B* changes from 2 to 4. Does the optimal solution change? Use the graphical solution procedure to find the new optimal solution.

d. The computer solution for the linear program in part (a) provides the following objective coefficient range information:

Variable	Objective Coefficient	Allowable Increase	Allowable Decrease
A	3.00000	3.00000	1.00000
B	2.00000	1.00000	1.00000

Use this objective coefficient range information to answer parts (b) and (c).

2. Consider the linear program in Problem 1. The value of the optimal solution is 27. Suppose that the right-hand side for constraint 1 is increased from 10 to 11.

a. Use the graphical solution procedure to find the new optimal solution.

b. Use the solution to part (a) to determine the dual value for constraint 1.

c. The computer solution for the linear program in Problem 1 provides the following right-hand-side range information:

Constraint	RHS Value	Allowable Increase	Allowable Decrease
1	10.00000	1.20000	2.00000
2	24.00000	6.00000	6.00000
3	16.00000	Infinite	3.00000

What does the right-hand-side range information for constraint 1 tell you about the dual value for constraint 1?

d. The dual value for constraint 2 is 0.5. Using this dual value and the right-hand-side range information in part (c), what conclusion can be drawn about the effect of changes to the right-hand side of constraint 2?

7. Investment Advisors, Inc., is a brokerage firm that manages stock portfolios for a number of clients. A particular portfolio consists of *U* shares of U.S. Oil and *H* shares of Huber Steel. The annual return for U.S. Oil is $3 per share and the annual return for Huber Steel is $5 per share. U.S. Oil sells for $25 per share and Huber Steel sells for $50 per share. The portfolio has $80,000 to be invested. The portfolio risk index (0.50 per share of U.S. Oil and 0.25 per share for Huber Steel) has a maximum of 700.

In addition, the portfolio is limited to a maximum of 1000 shares of U.S. Oil. The linear programming formulation that will maximize the total annual return of the portfolio is as follows:

$$
\begin{array}{llll}
\text{Max} & 3U + 5H & & \text{Maximize total annual return} \\
\text{s.t.} & & & \\
 & 25U + 50H \leq & 80{,}000 & \text{Funds available} \\
 & 0.50U + 0.25D \leq & 700 & \text{Risk maximum} \\
 & 1U \leq & 1000 & \text{U.S. Oil maximum} \\
 & U, H \geq 0 & &
\end{array}
$$

The computer solution of this problem is shown in Figure 3.14.

a. What is the optimal solution, and what is the value of the total annual return?

b. Which constraints are binding? What is your interpretation of these constraints in terms of the problem?

c. What are the dual values for the constraints? Interpret each.

d. Would it be beneficial to increase the maximum amount invested in U.S. Oil? Why or why not?

14. Digital Controls, Inc. (DCI), manufactures two models of a radar gun used by police to monitor the speed of automobiles. Model A has an accuracy of plus or minus 1 mile per hour, whereas the smaller model B has an accuracy of plus or minus 3 miles per hour. For the next week, the company has orders for 100 units of model A and 150 units of model B.

Although DCI purchases all the electronic components used in both models, the plastic cases for both models are manufactured at a DCI plant in Newark, New Jersey. Each model A case requires 4 minutes of injection-molding time and 6 minutes of assembly time. Each model B case requires 3 minutes of injection-

FIGURE 3.14 THE SOLUTION FOR THE INVESTMENT ADVISORS PROBLEM

Optimal Objective Value = 8400.00000

Variable	Value	Reduced Cost
U	800.00000	0.00000
H	1200.00000	0.00000

Constraint	Slack/Surplus	Dual Value
1	0.00000	0.09333
2	0.00000	1.33333
3	200.00000	0.00000

Variable	Objective Coefficient	Allowable Increase	Allowable Decrease
U	3.00000	7.00000	0.50000
H	5.00000	1.00000	3.50000

Constraint	RHS Value	Allowable Increase	Allowable Decrease
1	80000.00000	60000.00000	15000.00000
2	700.00000	75.00000	300.00000
3	1000.00000	Infinite	200.00000

molding time and 8 minutes of assembly time. For next week, the Newark plant has 600 minutes of injection-molding time available and 1080 minutes of assembly time available. The manufacturing cost is $10 per case for model A and $6 per case for model B. Depending upon demand and the time available at the Newark plant, DCI occasionally purchases cases for one or both models from an outside supplier in order to fill customer orders that could not be filled otherwise.

The purchase cost is $14 for each model A case and $9 for each model B case.

Management wants to develop a minimum cost plan that will determine how many cases of each model should be produced at the Newark plant and how many cases of each model should be purchased. The following decision variables were used to formulate a linear programming model for this problem:

AM = number of cases of model A manufactured
BM = number of cases of model B manufactured
AP = number of cases of model A purchased
BP = number of cases of model B purchased

The linear programming model that can be used to solve this problem is as follows:

$$\text{Min} \quad 10AM + 6BM + 14AP + 9BP$$

s.t.

$$\begin{array}{llll} 1AM + \quad + 1AP + & = & 100 & \text{Demand for model A} \\ \quad 1BM + \quad\quad 1BP & = & 150 & \text{Demand for model B} \\ 4AM + 3BM & \leq & 600 & \text{Injection molding time} \\ 6AM + 8BM & \leq & 1080 & \text{Assembly time} \end{array}$$

$$AM, BM, AP, BP \geq 0$$

The computer solution is shown in Figure 3.18.

a. What is the optimal solution and what is the optimal value of the objective function?

b. Which constraints are binding?

c. What are the dual values? Interpret each.

d. If you could change the right-hand side of one constraint by one unit, which one would you choose? Why?

FIGURE 3.18 THE SOLUTION FOR THE DIGITAL CONTROLS, INC., PROBLEM

Optimal Objective Value = 2170.00000

Variable	Value	Reduced Cost
AB	100.00000	0.00000
BM	60.00000	0.00000
AP	0.00000	1.75000
BP	90.00000	0.00000

Constraint	Slack/Surplus	Dual Value
1	0.00000	12.25000
2	0.00000	9.00000
3	20.00000	0.00000
4	0.00000	-0.37500

Variable	Objective Coefficient	Allowable Increase	Allowable Decrease
AB	10.00000	1.75000	Infinite
BM	6.00000	3.00000	2.33333
AP	14.00000	Infinite	1.75000
BP	9.00000	2.33333	3.00000

Constraint	RHS Value	Allowable Increase	Allowable Decrease
1	100.00000	11.42857	100.00000
2	150.00000	Infinite	90.00000
3	600.00000	Infinite	20.00000
4	1080.00000	53.33333	480.00000