Sheet 4-Chapter 5 PROBLEMS

Linear Programming: Simplex Method

1. Consider the following system of linear equations:

$$\begin{aligned} 3x_1 + x_2 \quad &= 6 \\ 2x_1 + 4x_2 + x_3 &= 12 \end{aligned}$$

- **a.** Find the basic solution with $x1 = 0$.
- **b.** Find the basic solution with $x2 = 0$.
- **c.** Find the basic solution with $x3 = 0$.
- **d.** Which of the preceding solutions would be basic feasible solutions for a linear program?

2. Consider the following linear program:

$$\begin{aligned} \text{Max} \quad & x_1 + 2x_2 \\ \text{s.t.} \quad & \\ & x_1 + 5x_2 \leq 10 \\ & 2x_1 + 6x_2 \leq 16 \\ & x_1, x_2 \geq 0 \end{aligned}$$

4. Consider the following linear programming problem:

$$\begin{aligned} \text{Max} \quad & 60x_1 + 90x_2 \\ \text{s.t.} \quad & \\ & 15x_1 + 45x_2 \leq 90 \\ & 5x_1 + 5x_2 \leq 20 \\ & x_1, x_2 \geq 0 \end{aligned}$$

- **a.** Write the problem in standard form.
- **b.** Develop the portion of the simplex tableau involving the objective function coefficients, the coefficients of the variables in the constraints, and the constants for the right-hand sides.

5. A partially completed initial simplex tableau is given:

Basis	c_B	x_1	x_2	s_1	s_2	
		5	9	0	0	
s_1	0	10	9	1	0	**90**
s_2	0	−5	3	0	1	**15**
z_j						
$c_j - z_j$						

a. Complete the initial tableau.
b. Which variable would be brought into solution at the first iteration?
c. Write the original linear program.

6. The following partial initial simplex tableau is given:

Basis	c_B	x_1	x_2	x_3	s_1	s_2	s_3	
		5	20	25	0	0	0	
		2	1	0	1	0	0	**40**
		0	2	1	0	1	0	**30**
		3	0	$-\frac{1}{2}$	0	0	1	**15**
z_j								
$c_j - z_j$								

a. Complete the initial tableau.
b. Write the problem in tableau form.
c. What is the initial basis? Does this basis correspond to the origin? Explain.
d. What is the value of the objective function at this initial solution?
e. For the next iteration, which variable should enter the basis, and which variable should leave the basis?
f. How many units of the entering variable will be in the next solution? Before making this first iteration, what do you think will be the value of the objective function after the first iteration?
g. Find the optimal solution using the simplex method.

8. Recall the problem for Par, Inc., introduced in Section 2.1. The mathematical model for this problem is restated as follows:

$$\begin{aligned}
\text{Max}\quad & 10x_1 + 9x_2 \\
\text{s.t.}\quad & \\
& \tfrac{7}{10}x_1 + 1x_2 \le 630 \quad \text{Cutting and dyeing} \\
& \tfrac{1}{2}x_1 + \tfrac{5}{6}x_2 \le 600 \quad \text{Sewing} \\
& 1x_1 + \tfrac{2}{3}x_2 \le 708 \quad \text{Finishing} \\
& \tfrac{1}{10}x_1 + \tfrac{1}{4}x_2 \le 135 \quad \text{Inspection and packaging} \\
& x_1, x_2 \ge 0
\end{aligned}$$

Where

x1 = number of standard bags produced

x2 = number of deluxe bags produced

- **a.** Use the simplex method to determine how many bags of each model Par should manufacture.
- **b.** What is the profit Par can earn with these production quantities?
- **c.** How many hours of production time will be scheduled for each operation?
- **d.** What is the slack time in each operation?

10. Solve the following linear program:

$$\begin{aligned}
\text{Max}\quad & 5x_1 + 5x_2 + 24x_3 \\
\text{s.t.}\quad & \\
& 15x_1 + 4x_2 + 12x_3 \le 2800 \\
& 15x_1 + 8x_2 \le 6000 \\
& x_1 + 8x_3 \le 1200 \\
& x_1, x_2, x_3 \ge 0
\end{aligned}$$

12. Suppose a company manufactures three products from two raw materials. The amount of raw material in each unit of each product is given.

Raw Material	Product A	Product B	Product C
I	7 lb	6 lb	3 lb
II	5 lb	4 lb	2 lb

If the company has available 100 pounds of material I and 200 pounds of material II, and if the profits for the three products are \$20, \$20, and \$15, respectively, how much of each product should be produced to maximize profits?

16. Set up the tableau form for the following linear program (do not attempt to solve):

$$\begin{aligned} \text{Min} \quad & 4x_1 + 5x_2 + 3x_3 \\ \text{s.t.} \quad & \\ & 4x_1 + 2x_3 \geq 20 \\ & 1x_2 - 1x_3 \leq -8 \\ & 1x_1 - 2x_2 = -5 \\ & 2x_1 + 1x_2 + 1x_3 \leq 12 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

23.	$\begin{aligned} \text{Max} \quad & 4x_1 + 8x_2 \\ \text{s.t.} \quad & \\ & 2x_1 + 2x_2 \leq 10 \\ & -1x_1 + 1x_2 \geq 8 \\ & x_1, x_2 \geq 0 \end{aligned}$	25.	$\begin{aligned} \text{Min} \quad & 1x_1 + 1x_2 \\ \text{s.t.} \quad & \\ & 8x_1 + 6x_2 \geq 24 \\ & 4x_1 + 6x_2 \geq -12 \\ & 2x_2 \geq 4 \\ & x_1, x_2 \geq 0 \end{aligned}$

30. Supersport Footballs, Inc., manufactures three kinds of footballs: an All-Pro model, a College model, and a High School model. All three footballs require operations in the following departments: cutting and dyeing, sewing, and inspection and packaging. The production times and maximum production availabilities are shown here.

	Production Time (minutes)		
Model	**Cutting and Dyeing**	**Sewing**	**Inspection and Packaging**
All-Pro	12	15	3
College	10	15	4
High School	8	12	2
Time available	300 hours	200 hours	100 hours

Current orders indicate that at least 1000 All-Pro footballs must be manufactured.

- **a.** If Supersport realizes a profit contribution of \$3 for each All-Pro model, \$5 for each College model, and \$4 for each High School model, how many footballs of each type should be produced? What occurs in the solution of this problem? Why?
- **b.** If Supersport can increase sewing time to 300 hours and inspection and packaging time to 150 hours by using overtime, what is your recommendation?